Exercise 13E: Solutions

1 **a** The point is in the first quadrant.

$$\begin{aligned} r &= \sqrt{1^2 + (\sqrt{3})^2} \\ &= \sqrt{1+3} = 2 \\ \cos\theta &= \frac{1}{2} \\ \theta &= \frac{\pi}{3} \end{aligned}$$

$$\therefore \quad 1 + \sqrt{3}i = 2 \text{ cis} \left(\frac{\pi}{3}\right)$$

b The point is in the fourth quadrant.

$$\begin{aligned} r &= \sqrt{1^2 + 1^2} \\ &= \sqrt{2} \\ \cos\theta &= \frac{1}{\sqrt{2}} \\ \theta &= -\frac{\pi}{4} \end{aligned}$$

$$\therefore \quad 1 - i = \sqrt{2} \text{ cis} \left(-\frac{\pi}{4}\right)$$

c The point is in the second quadrant.

$$\begin{aligned} r &= \sqrt{(2\sqrt{3})^2 + 2^2} \\ &= \sqrt{16} = 4 \\ \cos\theta &= \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2} \\ \theta &= \pi - \frac{\pi}{6} = \frac{5\pi}{6} \end{aligned}$$

$$\therefore \quad -2\sqrt{3} + 2i = 4 \text{ cis} \left(\frac{5\pi}{6}\right)$$

d The point is in the third quadrant.

$$\begin{aligned} r &= \sqrt{4^2 + 4^2} \\ &= \sqrt{32} = 4\sqrt{2} \\ \cos\theta &= -\frac{4}{4\sqrt{2}} = -\frac{1}{2} \\ \theta &= -\pi + \frac{\pi}{4} = -\frac{3\pi}{4} \end{aligned}$$

$$\therefore \quad -4 - 4i = 4\sqrt{2} \text{ cis} \left(-\frac{3\pi}{4}\right)$$

e The point is in the fourth quadrant.

$$\begin{aligned} r &= \sqrt{12^2 + 12^2 \times 3} \\ &= \sqrt{4 \times 144} = 24 \\ \cos\theta &= -\frac{12}{24} \\ &= -\frac{1}{2} \\ \theta &= -\frac{\pi}{3} \end{aligned}$$

$$\therefore \quad 12 - 12\sqrt{3}\, i = 24 \text{ cis} \left(-\frac{\pi}{3}\right)$$

f The point is in the second quadrant.

$$r=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}$$
$$=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$$
$$\cos\theta=-\frac{1}{2}\div\frac{1}{\sqrt{2}}$$
$$=-\frac{1}{2}\times\sqrt{2}=-\frac{1}{\sqrt{2}}$$
$$\theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$
$$\therefore\quad -\frac{1}{2}+\frac{1}{2}i=\frac{1}{\sqrt{2}}\operatorname{cis}\left(\frac{3\pi}{4}\right)$$

2 **a** $3\operatorname{cis}\frac{\pi}{2}=3\cos\frac{\pi}{2}+3i\sin\frac{\pi}{2}$
$=3i$

b $\sqrt{2}\operatorname{cis}\frac{\pi}{3}=\sqrt{2}\cos\frac{\pi}{3}+\sqrt{2}i\sin\frac{\pi}{3}$
$=\frac{\sqrt{2}}{2}+\frac{\sqrt{6}}{2}i$
$=\frac{\sqrt{2}}{2}(1+\sqrt{3}\,i)$

c $2\operatorname{cis}\frac{\pi}{6}=2\ \cos\frac{\pi}{6}+2i\sin\frac{\pi}{6}$
$=\sqrt{3}+i$

d $5\operatorname{cis}\frac{3\pi}{4}=5\cos\frac{3\pi}{4}+5i\sin\frac{3\pi}{4}$
$=-\frac{5}{\sqrt{2}}+\frac{5}{\sqrt{2}}i$
$=-\frac{5\sqrt{2}}{2}(1-i)$

e $12\operatorname{cis}\frac{5\pi}{6}=12\cos\frac{5\pi}{6}+12i\sin\frac{5\pi}{6}$
$=-6\sqrt{3}+6i$
$=-6(\sqrt{3}-i)$

f $3\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)=3\sqrt{2}\ \cos\left(-\frac{\pi}{4}\right)$
$+3\sqrt{2}i\sin\left(-\frac{\pi}{4}\right)$
$=3-3i$
$=3(1-i)$

g $5\operatorname{cis}\frac{4\pi}{3}=5\cos\frac{4\pi}{3}+5i\sin\frac{4\pi}{3}$
$=-\frac{5}{2}-\frac{5\sqrt{3}}{2}i$
$=-\frac{5}{2}(1+\sqrt{3}i)$

h $5\operatorname{cis}\left(-\frac{2\pi}{3}\right)=5\cos\left(-\frac{2\pi}{3}\right)$

$$+\,5i\sin\left(-\frac{2\pi}{3}\right)$$
$$=-\frac{5}{2}-\frac{5\sqrt{3}}{2}i$$
$$=-\frac{5}{2}(1+\sqrt{3}i)$$

3 $z_1\,z_2=r_1r_2\text{ cis }(\theta_1+\theta_2)$

a
$$\begin{aligned}\left(2\text{ cis }\frac{\pi}{6}\right)\cdot\left(3\text{ cis }\frac{\pi}{12}\right)&=6\text{ cis }\left(\frac{\pi}{6}+\frac{\pi}{12}\right)\\&=6\text{ cis }\frac{\pi}{4}\\&=6\cos\frac{\pi}{4}+6i\sin\frac{\pi}{4}\\&=\frac{6}{\sqrt{2}}+\frac{6}{\sqrt{2}}i\\&=3\sqrt{2}(1+i)\end{aligned}$$

b
$$\begin{aligned}\left(4\text{ cis }\frac{\pi}{12}\right)\cdot\left(3\text{ cis }\frac{\pi}{4}\right)&=12\text{ cis }\left(\frac{\pi}{12}+\frac{\pi}{4}\right)\\&=12\text{ cis }\frac{\pi}{3}\\&=12\cos\frac{\pi}{3}+12i\sin\frac{\pi}{3}\\&=6+6\sqrt{3}i\\&=6(1+\sqrt{3}i)\end{aligned}$$

c
$$\begin{aligned}\left(\text{cis }\frac{\pi}{4}\right)\cdot\left(5\text{ cis }\frac{5\pi}{12}\right)&=5\text{ cis }\left(\frac{\pi}{4}+\frac{5\pi}{12}\right)\\&=5\text{ cis }\frac{2\pi}{3}\\&=5\cos\frac{2\pi}{3}+5i\sin\frac{2\pi}{3}\\&=-\frac{5}{2}+\frac{5\sqrt{3}}{2}i\\&=-\frac{5}{2}(1-\sqrt{3}i)\end{aligned}$$

d
$$\begin{aligned}\left(12\text{ cis }\left(-\frac{\pi}{3}\right)\right)\cdot\left(3\text{ cis }\frac{2\pi}{3}\right)&=36\text{ cis }\left(-\frac{\pi}{3}+\frac{2\pi}{3}\right)\\&=36\text{ cis }\frac{\pi}{3}\\&=36\cos\frac{\pi}{3}+36i\sin\frac{\pi}{3}\\&=18+18\sqrt{3}i\\&=18(1+\sqrt{3}i)\end{aligned}$$

e
$$\begin{aligned}\left(12\text{ cis }\frac{5\pi}{6}\right)\cdot\left(3\text{ cis }\frac{\pi}{2}\right)&=36\text{ cis }\left(\frac{5\pi}{6}+\frac{\pi}{2}\right)\\&=36\text{ cis }\frac{4\pi}{3}\\&=36\cos\frac{4\pi}{3}+36i\sin\frac{4\pi}{3}\\&=-18-18\sqrt{3}i\\&=-18(1+\sqrt{3}i)\end{aligned}$$

f $$\left(\sqrt{2}\text{ cis }\pi\right)\cdot\left(\sqrt{3}\text{ cis}\left(-\frac{3\pi}{4}\right)\right)=\sqrt{6}\text{ cis}\left(\pi-\frac{3\pi}{4}\right)$$
$$=\sqrt{6}\text{ cis }\frac{\pi}{4}$$
$$=\sqrt{6}\cos\frac{\pi}{4}+\sqrt{6}i\sin\frac{\pi}{4}$$
$$=\sqrt{3}+\sqrt{3}i$$
$$=\sqrt{3}(1+i)$$

g $$\frac{10\text{ cis }\frac{\pi}{4}}{5\text{ cis }\frac{\pi}{12}}=\frac{10}{5}\text{ cis}\left(\frac{\pi}{4}-\frac{\pi}{12}\right)$$
$$=2\text{ cis }\frac{\pi}{6}$$
$$=2\cos\frac{\pi}{6}+2i\sin\frac{\pi}{6}$$
$$=\sqrt{3}+i$$

h $$\frac{12\text{ cis}\left(-\frac{\pi}{3}\right)}{3\text{ cis }\frac{2\pi}{3}}=\frac{12}{3}\text{ cis}\left(-\frac{\pi}{3}-\frac{2\pi}{3}\right)$$
$$=4\text{ cis }(-\pi)$$
$$=4\cos\ (-\pi)+4i\sin(-\pi)$$
$$=-4+0=-4$$

i $$\frac{12\sqrt{8}\text{ cis }\frac{3\pi}{4}}{3\sqrt{2}\text{ cis }\frac{\pi}{12}}=\frac{12\sqrt{8}}{3\sqrt{2}}\text{ cis}\left(\frac{3\pi}{4}-\frac{\pi}{12}\right)$$
$$=8\text{ cis }\frac{2\pi}{3}$$
$$=8\cos\frac{2\pi}{3}+8i\sin\frac{2\pi}{3}$$
$$=-4+4\sqrt{3}i$$
$$=-4(1-\sqrt{3}i)$$

j $$\frac{20\text{ cis}\left(-\frac{\pi}{6}\right)}{8\text{ cis }\frac{5\pi}{6}}=\frac{20}{8}\text{ cis}\left(-\frac{\pi}{6}-\frac{5\pi}{6}\right)$$
$$=\frac{5}{2}\text{ cis }(-\pi)$$
$$=\frac{5}{2}\cos(-\pi)+\frac{5}{2}i\sin\ (-\pi)$$
$$=-\frac{5}{2}+0$$
$$=-\frac{5}{2}$$